

# Hadron Structure Functions within a Chiral Quark Model\*

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We outline a consistent regularization procedure to compute hadron structure functions within bosonized chiral quark models. We impose the Pauli–Villars scheme, which reproduces the chiral anomaly, to regularize the bosonized action. We derive the Compton amplitude from this action and utilize the Bjorken limit to extract structure functions that are consistent with the scaling laws and sum rules of deep inelastic scattering.

## 1. THE CHIRAL MODEL

The bosonized action of chiral quark models can be cast in the form

$$\mathcal{A}[S, P] = -iN_C \text{Tr}_\Lambda \log [i\cancel{\partial} - (S + i\gamma_5 P)] - \frac{1}{4G} \int d^4x \text{tr } \mathcal{V}(S, P). \quad (1)$$

Here  $\mathcal{V}$  is a local potential respectively for scalar and pseudoscalar fields  $S$  and  $P$  which are matrices in flavor space. For example, in the Nambu–Jona–Lasinio (NJL) model [1] one has  $\mathcal{V} = S^2 + P^2 + 2\hat{m}_0(S + iP)$ . From the gap-equation we obtain the VEV,  $\langle S \rangle = m$  which parameterizes the dynamical chiral symmetry breaking. The regularization of the quadratically divergent quark loop is indicated by the cut-off  $\Lambda$ . We adjust its value as well as the coupling constant  $G$  and the current quark mass  $\hat{m}_0$  to fit the phenomenological meson parameters  $m_\pi$  and  $f_\pi$ , leaving only a single free parameter, the constituent quark mass,  $m$ . An essential feature of these models is that the derivative term in (1) is formally identical to that of a non-interacting quark model. Hence the current operator is given as  $J^\mu = \bar{q} \mathcal{Q} \gamma^\mu q$ , with  $\mathcal{Q}$  being a flavor matrix. We compute expectation values of currents by introducing pertinent sources in the bosonized action and taking appropriate derivatives.

The major concern in regularizing the functional (1) is to maintain the chiral anomaly. We achieve this goal by splitting this functional into  $\gamma_5$ -even and odd pieces and only regularize the former. The  $\gamma_5$ -odd part turns out to be conditionally finite.

Details of this presentation are published in [2]. For related work see refs [3–5].

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## 2. REGULARIZATION OF THE COMPTON TENSOR

DIS off hadrons is parameterized by the hadronic tensor  $W^{\mu\nu}(q)$  with  $q$  being the momentum transmitted to the hadron by the photon.  $W^{\mu\nu}(q)$  is obtained from the hadron matrix element of the commutator  $[J^\mu(\xi), J^\nu(0)]$ . In bosonized quark models we find it convenient to start from the absorptive part of the forward virtual Compton amplitude

$$T^{\mu\nu}(q) = \int d^4\xi e^{iq\cdot\xi} \langle p, s | T (J^\mu(\xi) J^\nu(0)) | p, s \rangle \quad \text{and} \quad W^{\mu\nu}(q) = \frac{1}{2\pi} \Im (T^{\mu\nu}(q)). \quad (2)$$

(We denote the momentum of the hadron by  $p$  and eventually its spin by  $s$ .) The advantage is that the time-ordered product is unambiguously obtained from the regularized action

$$T (J^\mu(\xi) J^\nu(0)) = \frac{\delta^2}{\delta v_\mu(\xi) \delta v_\nu(0)} \text{Tr}_\Lambda \log [i\cancel{\partial} - (S + i\gamma_5 P) + \mathcal{Q}\cancel{\psi}] \Big|_{v_\mu=0}. \quad (3)$$

In order to extract the leading twist pieces of the structure functions, we study  $W^{\mu\nu}(q)$  in the Bjorken limit:  $q^2 \rightarrow -\infty$  with  $x = -q^2/p \cdot q$  fixed.

We now have to specify the regularization of the functional trace in (3). We define

$$i\mathbf{D} = i\cancel{\partial} - (S + i\gamma_5 P) + \cancel{\psi}\mathcal{Q} \quad \text{and} \quad i\mathbf{D}_5 = -i\cancel{\partial} - (S - i\gamma_5 P) - \cancel{\psi}\mathcal{Q} \quad (4)$$

and separate the functional trace into (un-)regularized  $\gamma_5$ -even (odd) pieces,

$$\begin{aligned} \text{Tr}_\Lambda \log [i\cancel{\partial} - (S + i\gamma_5 P) + \mathcal{Q}\cancel{\psi}] &= -i \frac{N_C}{2} \sum_{i=0}^2 c_i \text{Tr} \log [-\mathbf{D}\mathbf{D}_5 + \Lambda_i^2 - i\epsilon] \\ &\quad - i \frac{N_C}{2} \text{Tr} \log [-\mathbf{D}(\mathbf{D}_5)^{-1} - i\epsilon]. \end{aligned} \quad (5)$$

With the conditions  $c_0 = 1$ ,  $\Lambda_0 = 0$ ,  $\sum_{i=0}^2 c_i = 0$  and  $\sum_{i=0}^2 c_i \Lambda_i^2 = 0$  the double Pauli-Villars regularization renders the functional in (3) finite.

## 3. PION STRUCTURE FUNCTION

DIS off pions is characterized by a single structure function,  $F(x)$ . For its computation we have to specify the pion matrix element in the Compton amplitude (2). Whence we introduce the pion field  $\vec{\pi}$  via<sup>3</sup>

$$S + iP\gamma_5 = m (U)^{\gamma_5} = m \exp \left( i \frac{g}{m} \gamma_5 \vec{\pi} \cdot \vec{\tau} \right). \quad (6)$$

Expanding (5,6) to linear and quadratic order in  $\vec{\pi}$  and  $v_\mu$ , respectively yields the proper result for the anomalous decay  $\pi^0 \rightarrow \gamma\gamma$ . In turn we obtain the Compton amplitude for virtual pion-photon scattering by expanding (5,6) to second order in both,  $\vec{\pi}$  and  $v_\mu$ . Due to the separation into  $\mathbf{D}$  and  $\mathbf{D}_5$  this calculation differs from the evaluation of the ‘hand-bag’ diagram because isospin violating dimension-five operators emerge. Fortunately all isospin violating pieces cancel yielding

$$F(x) = \frac{5}{9} (4N_C g^2) \frac{d}{dm_\pi^2} \left\{ m_\pi^2 \sum_{i=0}^2 c_i \frac{d^4 k}{(2\pi)^{4i}} [-k^2 - x(1-x)m_\pi^2 + m^2 + \Lambda_i^2 - i\epsilon]^{-2} \right\}. \quad (7)$$

<sup>3</sup>The coupling  $g$  and the constituent quark mass  $m$  are related by the pion decay constant. In the chiral limit the relation is linear  $m = g f_\pi$ .

The cancellation of the isospin violating pieces is a feature of the Bjorken limit: insertions of the pion field on the propagator carrying the infinitely large photon momentum can be safely ignored. Furthermore this propagator can be taken to be the one for non-interacting massless fermions. This implies that the Pauli–Villars cut-offs can be omitted for this propagator leading to the desired scaling behavior of the structure function.

#### 4. NUCLEON STRUCTURE FUNCTIONS

In the bosonized chiral quark model baryons emerge as solitons of the meson fields [6]. We parameterize the soliton by

$$U(\vec{x}, t) = A(t) \exp(i\vec{\tau} \cdot \hat{r} \Theta(r)) A^\dagger(t), \quad (8)$$

with the chiral angle  $\Theta(r)$  being determined from the stationary condition for constant  $A$ . Subsequently we quantize the collective coordinates  $A$  to generate nucleon states.

As argued above we take the quark propagator with the infinite photon momentum to be free and massless. Thus, it is sufficient to differentiate

$$\begin{aligned} & \frac{N_C}{4i} \sum_{i=0}^2 c_i \text{Tr} \left\{ \left( -\mathbf{D}^{(\pi)} \mathbf{D}_5^{(\pi)} + \Lambda_i^2 \right)^{-1} \left[ \mathcal{Q}^2 \psi (\not{\partial})^{-1} \not{\psi} \mathbf{D}_5^{(\pi)} - \mathbf{D}^{(\pi)} (\psi (\not{\partial})^{-1} \not{\psi})_5 \mathcal{Q}^2 \right] \right\} \\ & + \frac{N_C}{4i} \text{Tr} \left\{ \left( -\mathbf{D}^{(\pi)} \mathbf{D}_5^{(\pi)} \right)^{-1} \left[ \mathcal{Q}^2 \psi (\not{\partial})^{-1} \not{\psi} \mathbf{D}_5^{(\pi)} + \mathbf{D}^{(\pi)} (\psi (\not{\partial})^{-1} \not{\psi})_5 \mathcal{Q}^2 \right] \right\}, \end{aligned} \quad (9)$$

with respect to the photon field  $v_\mu$  as in eq (3). We have introduced the  $(\dots)_5$  description

$$\gamma_\mu \gamma_\rho \gamma_\nu = S_{\mu\rho\nu\sigma} \gamma^\sigma - i\epsilon_{\mu\rho\nu\sigma} \gamma^\sigma \gamma^5 \quad \text{and} \quad (\gamma_\mu \gamma_\rho \gamma_\nu)_5 = S_{\mu\rho\nu\sigma} \gamma^\sigma + i\epsilon_{\mu\rho\nu\sigma} \gamma^\sigma \gamma^5 \quad (10)$$

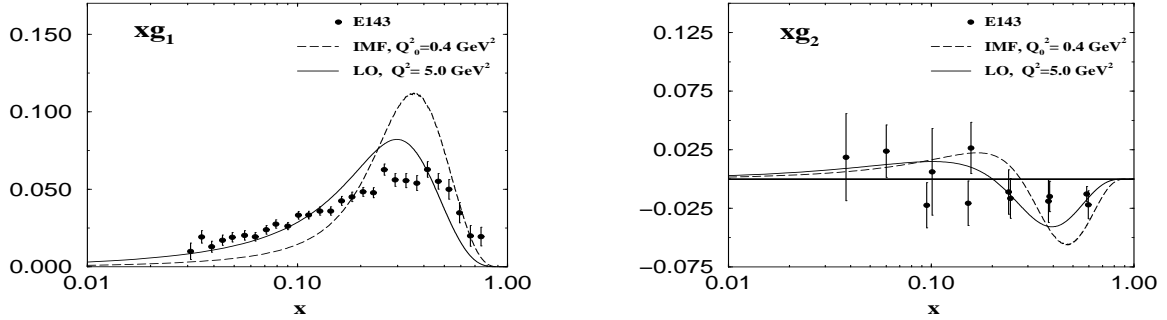
to account for the unconventional appearance of axial sources in  $\mathbf{D}_5$  [2]. Upon substituting (8) into (9) and computing the functional trace, using a basis of quark states obtained from the Dirac Hamiltonian in the background of  $U(\vec{x}, t)$ , we find analytical results for the structure functions. By construction their regularization is consistent with the chiral anomaly. We refer to [2] for detailed formulae and the explicit verification of sum rules. Here we simply report the important result that the structure function entering the Gottfried sum rule is related to the  $\gamma_5$ -odd piece of the action and hence does not undergo regularization. This is surprising because in the parton model this structure function differs from the one associated with the Adler sum rule only by the sign of the anti-quark distribution. The latter structure function, however, gets regularized, in agreement with the quantization rules for the collective coordinates. As we have consistently implemented the regularization at the level of the bosonized action this demonstrates that in effective models these structure functions are quite different from constituent quark distributions and in their description one has to go beyond identifying degrees of freedom.

#### 5. NUMERICAL RESULTS FOR THE POLARIZED NUCLEON STRUCTURE FUNCTIONS

Unfortunately numerical results for the full structure functions, *i.e.* including the properly regularized vacuum piece are not yet available. However, we have verified that in the Pauli–Villars regularization the axial charges are saturated to 95% or more by their valence quark contributions once the self-consistent soliton is substituted. This provides

sufficient justification to adopt the valence quark contribution to the polarized structure functions as a reliable approximation [3]. In Fig. 1 we compare the model predictions for the linearly independent polarized structure functions to experimental data [9].

Fig. 1: Model predictions for the polarized proton structure functions  $xg_1$  (left panel) and  $xg_2$  (right panel). The curves labeled ‘RF’ denote the results as obtained from the valence quark contribution to (9). These undergo a projection to the infinite momentum frame ‘IMF’ [7] and a leading order ‘LO’ DGLAP evolution [8]. Data are from SLAC–E143 [9].



The evolution of the structure function  $g_2$  to the momentum scale of the experiments requires the separation into twist–2 and twist–3 components [8]. We observe that the model results for the polarized structure functions, which we argued to have reliably approximated, agree reasonably well with the experimental data. This encourages future work in this direction.

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